PACKAGE OF ALGORITHMS FOR A POSTERIORI DETERMINATION OF RADAR SYSTEMATIC ERRORS FOR SEVERAL RADARS *

D. A. Bedin[1], A. G. Ivanov[2], A. A. Fedotov[3]

N.N. Krasovskii Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences, Ekaterinburg, Russia, e-mail: bedin@imm.uran.ru

S. A. Ganebny[4]

"NITA" LLC, St. Petersburg, Russia, e-mail: gsa@nita.ru

Abstract

Key words: systematic error, radar, parametric estimation

We consider a problem of estimation of the radar systematic errors (or radar biases). These errors are nonrandom distortions in measurements, which depend on location of an aircraft. Three algorithms are suggested to solve the problem. Each algorithm is based on its own assumptions and is effective under them. A program package was elaborated that includes all the algorithms. Correctness of the work of these algorithms was checked on model and real radar data.

Introduction

Radar measurements are subjected to errors like measurements of other sensors. There are two types of radar errors: one type of errors has a probabilistic nature, *i.e.*, this is the random noise, and another type is non-random distortions, namely, the systematic errors. The processing of the real data showed that, in general, the systematic errors depend on the relative mutual location of an observed aircraft and the radar, and they are stable in time. A vector field is a correct mathematical structure for description of the systematic errors: a shift vector for measurements depends on spatial location of an aircraft.

Determination of the systematic errors in radar data and their following correction are important operations for stable work of the air traffic control systems [1]. Especially, it concerns multisensor tracking [2]. There are many methods for solving this problem. One of them consists in joint processing of the radar and ADS-B data. But the authors consider another variant where the determination is only based on the radar data without any external information.

Measurement model

Let m be a total number of radars. Consider the observation equation for description of the measurement process of the radar i in a Cartesian coordinate system; for example, it can be the geocentric system. Let $x(t)$ be the vector of the current aircraft location at the instant t. Denote a vector of the radar measurement as z_i; the quantity s_i is a shift vector that describes influence of the systematic errors on a measurement; the vector w_i denotes a shift related to the random errors. Then the observation equations are

$$z_i(t) = x(t) + s_i(x(t)) + w_i(t), \qquad i = 1,\ldots,m. \tag{1}$$

The radar does not measure the location of an aircraft directly in the Cartesian coordinate system. Instead, it gives a triple (z_i^r, z_i^α, z^h) at the output where z_i^r, z_i^α are its own measurements of the slant range to the target and the target azimuth, and z^h is the altitude measurement that is received directly from the aircraft (in the case of so-called secondary radar). The mapping $d_i:\ (z_i^r, z_i^\alpha, z^h) \mapsto z_i$ is a one-to-one and invertible mapping everywhere outside some neighborhood of the radar. Often, it is convenient to consider the observation equations in the terms of the direct radar measurements:

$$z_i^r(t) = r_i(x(t)) + \Delta_i^r(x(t)) + w_i^r(t), \qquad z_i^\alpha(t) = \alpha_i(x(t)) + \Delta_i^\alpha(x(t)) + w_i^\alpha(t), \qquad z^h(t) = h(x(t)) + w^h(t). \tag{2}$$

[1] MSc, Researcher.
[2] Main developer.
[3] Cand. Sci. Phys.-Math., Researcher.
[4] Cand. Sci. Phys.-Math., Engineer and developer.

Here the variables Δ_i^r, Δ_i^α are the systematic errors in the slant range and azimuth. There are random errors in all measurement channels. They are denoted by w_i^r, w_i^α, w^h. Symbols $r_i(x)$, $\alpha_i(x)$ denote the slant range from the target to the radar i and its azimuth, respectively; $h(x)$ is the aircraft altitude above the Earth surface.

If the altitude $h(x)$ is not measured (for example, by the so-called primary radars), the measurement (z_i^r, z_i^α) will not uniquely correspond to some exact location in 3D space. Nevertheless, in the most such events, there are measurements of this aircraft by other radars with the presence of the altitude. So, it provides an opportunity to resolve this uncertainty.

A popular model of the radar systematic errors is the model of so-called the constant systematic errors in range and azimuth. In this case, the values Δ_i^r, Δ_i^α do not depend on location x of an aircraft.

The problem of estimation of radar systematic errors

Three algorithms were elaborated for the problem of estimating the radar systematic error: the parametric estimation algorithm, the non-parametric estimation algorithm, and the non-parametric estimation algorithm for work in the presence of distortions in measurement instants. Each of them works effectively under only its own assumption, but all the algorithms use the information redundancy that appears when several radars observe one aircraft.

1. In the parametric estimation algorithm, it is supposed that the unknown distortion of the radar data can be present as a combination of several known functions. These functions give the structure of the systematic error vector field; there are some unknown parameters for fitting this model to the data. For the systematic error in range and azimuth, we describe such a field as follows:

$$\forall x \in R^3, i = 1,\ldots,m \qquad \Delta_i^r(x) = \boldsymbol{D}_i^r(x,\vartheta), \quad \Delta_i^\alpha(x) = \boldsymbol{D}_i^\alpha(x,\vartheta) \,. \tag{3}$$

Here, the symbol ϑ denotes the column vector of unknown parameters. For the Cartesian coordinate system, we use another description:

$$\forall x \in R^3, i = 1,\ldots,m \qquad s_i(x) = \boldsymbol{S}_i(x,\vartheta) \,. \tag{4}$$

The simplest example of the parametric model is the model of the constant systematic errors in the range and azimuth. In this case, equation (3) turns into expression

$$\forall x \in R^3, i = 1,\ldots,m \qquad \Delta_i^r(x) = \Delta_i^r = \text{const}, \quad \Delta_i^\alpha(x) = \Delta_i^\alpha = \text{const}, \quad \vartheta = \left\{ (\Delta_i^r, \Delta_i^\alpha) \right\}_{i=1}^m .$$

In this algorithm, each aircraft trajectory is analyzed separately and gives the individual estimate for parameters in the model of the systematic errors. The evaluation procedure consists of minimization of the mean square error. The main version of the algorithm uses nonlinear mapping d_i and physical constraints on the aircraft motion parameters. These circumstances lead to using numerical procedures in optimization. We apply a method that combines the gradient descent in some variables with the Hooke-Jeeves pattern search method (with additional using of the Monte-Carlo method) in other directions. In the case of the linearized model of observation (*i.e.*, when the mapping d_i is linearized), the minimum point of the functional can be evaluated analytically. A version of the algorithm with such a simplification was elaborated.

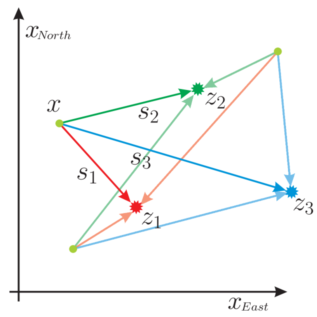
In the second stage of the algorithm, the set of the individual estimates $\{\vartheta\}$ for all aircraft is analyzed by means of a statistical procedure to give the final estimate $\hat{\vartheta}$ of parameters ϑ and the estimates for the systematic error fields $\hat{\Delta}_i^r(\cdot)$, $\hat{\Delta}_i^\alpha(\cdot)$, $\hat{s}_i(\cdot)$. It should be noted that the final estimates for the fields do not have to be in the same parametric form

$$\forall x \in R^3, i = 1,\ldots,m \qquad \hat{\Delta}_i^r(x) = \boldsymbol{D}_i^r(x,\hat{\vartheta}) \,, \; \hat{\Delta}_i^\alpha(x) = \boldsymbol{D}_i^\alpha(x,\hat{\vartheta}) \,, \; \hat{s}_i(x) = \boldsymbol{S}_i(x,\hat{\vartheta}) \,.$$

For example, the field estimate $\hat{\Delta}_i^r(x)$, $\hat{\Delta}_i^\alpha(x)$ with piecewise-constant dependence on x shows good agreement with the real data. This estimate is obtained by the averaging of individual estimates in small geographic areas with taking into account the number of measurements in every area.

2. The non-parametric estimation algorithm is related to the case when the model of the systematic errors in the form (3) or (4) is unknown, or the results of the algorithm from the previous section cannot adequately correct the measurements of radars. The non-parametric estimation algorithm does not use any prescribed model of the systematic errors. There are two stages of this algorithm: the first stage consists of evaluation of uncertainty sets, the second one is in selection of a single-valued function as the final estimate.

The uncertainty set is a set of "mean" vectors in the shift space for all variants of the systematic errors, which are compatible with the radar measurements at specific spatial location. Now we try to explain this fact by a picture. In the Figure there are measurements z_1, z_2, z_3 of three radars at the same time instant. Even in the case of measurements without random noise, there are many variants of the true location of the aircraft. Three of them are shown in the figure. If we do not know any *a priori* information about values of the systematic error shifts s_i, the true location x of the aircraft can be anywhere near the measurements z_1, z_2, z_3. However, the only one variant of the shifts s_1, s_2, s_3 corresponds to every location x. If the measurements have random noise, we can consider the correspondence between x and the average values of s_1, s_2, s_3. Since x belongs to 2D plane in 3D space (this plane corresponds to the altitude h of the aircraft), we get an 2D affine manifold S in the space of vectors $s = \left[s_1^T \quad s_2^T \quad s_3^T \right]^T$ by choosing arbitrary x and corresponding average s_1, s_2, s_3. The manifold S is named as the *uncertainty set*.



In the algorithm, the uncertainty sets are constructed for the specific mesh of "small" geographic areas. Every uncertainty set S exploits only local radar measurements from its own small area. The procedure of construction of the uncertainty set uses radar measurements at the same instant; but, actually, different radars never measure location of the aircraft at the same instant. For this reason, we make the "artificial" measurements approximating every radar track by means of polyline. Every uncertainty set S is attached to the center of corresponding "small" area. So, we can consider the sets as a multi-valued function $S(x)$ that corresponds to all variants of the systematic error fields, which satisfactory fit the data.

The second stage of the non-parametric estimation algorithm is related to searching a single-valued function $s(\cdot)$ of the systematic errors. In this searching, a more "weak" knowledge than parametric models (3) or (4) can be used for the systematic error functions. For example, we can require that the functional of the integral spatial slope of the function $s(\cdot)$ should have small values. Presently, this algorithm uses the functional of the mean-square variation.

3. The third algorithm has the same stages and similar constructions as the non-parametric estimation algorithm in the Section 2; but it does not use the instants of radar measurements. The uncertainty sets are constructed in a special way. Firstly, all aircraft trajectories are divided into sections of the straight line motion. Then for each section, the direction of the motion is estimated by means of the principal component analysis method [3]. The shift vectors of the systematic errors have components along this direction of motion, but their determination are not possible since distortions in measurement instants are equivalent to spatial shift of the measurements along this direction. All variants of the systematic error shifts along all remaining components are combined into uncertainty sets. Other constructions of this algorithm completely repeat corresponding constructions of the previous non-parametric estimation algorithm.

Conclusions

The program package is implemented in MATLAB. Correctness of the work of the algorithms is checked on the model and real radar data.

Investigations were performed under collaboration with NITA LLC. The reported study was supported by the Russian Foundation for Basic Research, project no. 15-01-07909.

References

1. **Bar-Shalom, Y.**, Exact multisensor dynamic bias estimation with local tracks / X. Lin, Y. Bar-Shalom, T. Kirubarajan , IEEE Transactions on Aerospace and Electronic Systems, 2004, vol. 40, no. 2, pp. 576–590.
2. **Siegelmann, H. T.,** Sensor Registration Using Neural Networks / H. Karniely, H. T. Siegelmann **,** IEEE Transactions on Aerospace and Electronic Systems, 2000, vol. 36, no. 1, pp. 85–101.
3. **Jolliffe, I. T.**, Principal Component Analysis. New York: Springer-Verlag. 2002. P. 405.