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Coprocessing of Data from Several Radars for Determination of Systematic Errors in Azimuth and Range

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The paper describes three approaches for solving the problem of estimation of systematic errors in radar measurements obtained from several overlapping observation domains. In the proposed approaches, neither one of the radars could be considered as a reference one. The systematic errors are determined by the processing of measurements of the radars due to informational redundancy in the radar data. The proposed methods can be used in computational information complexes of the air traffic control systems.

I. Introduction

Big air traffic control (ATC) centers handle data incoming from a number of radars. In the coprocessing of these data, it is important to determine systematic errors in radar measurements of aircraft position. These errors can influence on the accuracy and the consistency of working algorithms in ATC systems. Contemporary ATC zones have extensive size; so, all aircrafts are observed by a number of radars (up to 10) simultaneously. This information redundancy makes possible to evaluate the systematic errors.

A. Observation model

Usually, a radar measures two values that characterize an aircraft position. The first one is a slant range r from an aircraft to the radar, and the second one is an azimuth α, i.e., the angle between the radar direction to the North and the horizontal projection of the "radar–aircraft" beam (Fig. 1). In addition, a secondary surveillance radar (SSR) receives the altitude h from the aircraft. Generally, the measurements of the altitude have high accuracy and similar values at close instants among tracks of various radars. It is easy to convert a triple (r, α, h) of the radar measurement into geocentric and geographic coordinates.

Let m be a total number of radars that observe an aircraft A. Consider the observation process of the radar with identifier i (this symbol will be present in subscripts of the corresponding variables). Let $x(t)$ be a vector of the current aircraft position at the instant t, and $r_i(x)$, $\alpha_i(x)$ be functions that return the range from the object to the

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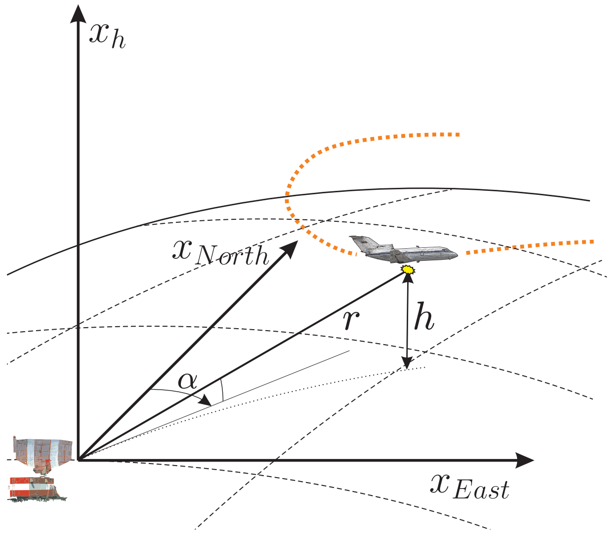


Figure 1. Observation of an aircraft by a radar

radar i and its azimuth, respectively. A function $h(x)$ returns the aircraft altitude above the Earth surface. Then the observation equations are

$$\begin{aligned} z_i^r(A,t) &= r_i(x(t)) + \Delta_i^r(x(t)) + w_i^r(t), \\ z_i^\alpha(A,t) &= \alpha_i(x(t)) + \Delta_i^\alpha(x(t)) + w_i^\alpha(t), \\ z^h(A,t) &= h(x(t)) + w^h(x(t)). \end{aligned} \tag{1}$$

Here, z_i^r, z_i^α, z^h are measurements of the range, azimuth, and altitude, respectively. Symbols in the parenthesis mean that the aircraft A is under observation at the instant t. Further on, these symbols will be omitted if this does not lead to ambiguity. The variables Δ_i^r, Δ_i^α denote the systematic errors in the range and azimuth, which are non-random distortions depending on the aircraft position x w.r.t. the radar i. Also, there are random errors in all measurement channels. In (1), they are denoted by w_i^r, w_i^α, w^h. In the range and azimuth channels, the random errors have an adequate probabilistic description. The expected values of w_i^r and w_i^α are equal to zero and their covariance matrices are known. For any distinct instants t_1 and t_2, the random variables $w_i^r(t_1)$, $w_i^r(t_2)$, $w_i^\alpha(t_1)$, and $w_i^\alpha(t_2)$ are mutually independent and do not depend on the aircraft position x. Errors in the altitude channel have a more complicated character. This fact is connected with the discrete nature of the altitude data. But in practice, the variable w^h has a small magnitude and does not affect significantly on further evaluation.

Using the geocentric coordinate system, equation (1) can be written as

$$z_i(A,t) = x(t) + s_i(x(t)) + w_i(t, x(t)). \tag{2}$$

In this equation, the symbol z_i denotes a vector that corresponds to the triple (z_i^r, z_i^α, z^h); the quantity s_i is a shift vector that describes the effect of the systematic error on a measurement; the vector w_i denotes a shift related to the random errors.

B. Problem statement

If an aircraft moves in a number of overlapped radar observation domains and each radar has its own systematic errors $(\Delta_i^r(x),\ \Delta_i^\alpha(x))$, the tracks of this aircraft from various

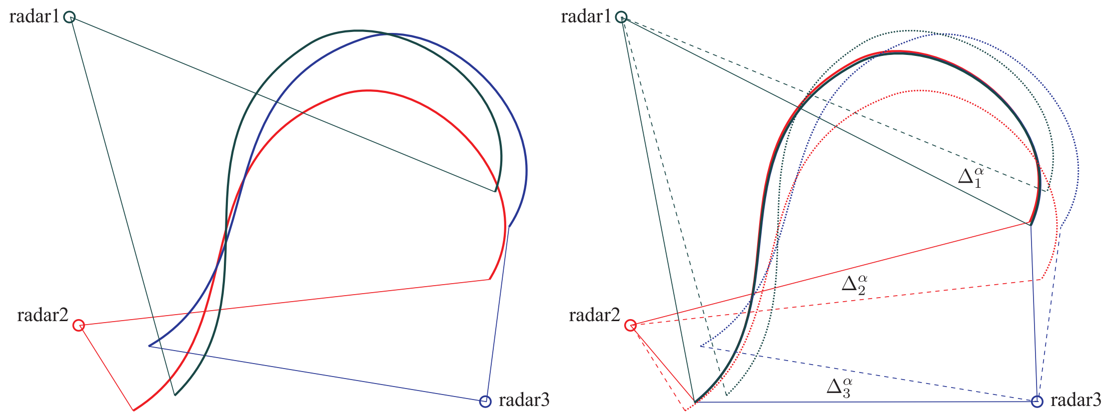


Figure 2. The tracks of the three radars that observe one aircraft (the scheme). The left side shows the original tracks. The right side shows those after corrections of the azimuth systematic errors Δ_i^α

radars will differ from each other (Fig. 2). In the case when additional reference measurements are present, the systematic errors of each radar will be easily determined by comparison of the reference and radar data. But such an additional external information (e.g., from ADS-B system) about trajectories of aircrafts is not always easy to get in practice. In this work, we deal with the algorithms that use only radar information.

The determination of the systematic errors can be considered as a problem of finding corrections that align all the radar data. A correction vector for the radar i can be taken as $-\hat{s}_i(x)$, where $\hat{s}_i(x)$ is an estimate of the shift vector $s_i(x)$. It is desirable that the aligned tracks of one aircraft from various radars would look like one track after correction shifts. The aligning problem can be solved if the estimations $\hat{s}_i(x)$ are close to the true value of $s_i(x)$ or, equivalently, the estimations $\hat{\Delta}_i^r(x)$, $\hat{\Delta}_i^\alpha(x)$ are close to the true values of $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$. So, we try to find "good" estimations for systematic errors.

In the algorithms of sections II and III, we suggest that the functions $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$ (or $s_i(x)$) belong to some parametric class of functions:

$$\forall x \in \mathbb{R}^{\mathbf{3}},\ \forall i = \overline{1, m} \qquad \Delta_i^r(x) = \mathcal{D}^r(x, \theta), \quad \Delta_i^\alpha(x) = \mathcal{D}^\alpha(x, \theta),$$

where θ is the vector of all unknown parameters of the model. In this case, we try to find estimates in the same class:

$$\forall i = \overline{1, m} \qquad \hat{\Delta}_i^r(\cdot) = \mathcal{D}^r(\cdot, \hat{\theta}), \quad \hat{\Delta}_i^\alpha(\cdot) = \mathcal{D}^\alpha(\cdot, \hat{\theta}).$$

This leads to an estimation of the unknown parameter θ through some criterion that depends on the quality of the track aligning after correction by means of $\hat{\Delta}_i^r(x)$, $\hat{\Delta}_i^\alpha(x)$. This procedure provides a small deviation of

$$\mathbf{E}\left\{\|\hat{\theta} - \theta\|^{\mathbf{2}}\right\}$$

and, consequently, small deviations of the estimations from true values

$$\mathbf{E}\left\{\|\hat{\Delta}_i^r(x) - \Delta_i^r(x)\|^{\mathbf{2}}\right\}, \quad \mathbf{E}\left\{\|\hat{\Delta}_i^\alpha(x) - \Delta_i^\alpha(x)\|^{\mathbf{2}}\right\} \quad (\forall x \in \mathbb{R}^{\mathbf{3}}).$$

The most simple is the variant where the unknown parameter θ consists of all constant systematic errors (in the range and azimuth) for all radars.

The algorithm in the section IV uses a non-parametric technique without any preliminary defined models of $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$. It turned out that, in any small region of the observation area, measurements enable to estimate some linear combinations of all average values of the functions $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$ in that region. The linear combinations can be estimated without any additional assumptions on the structure of the functions $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$ themselves. Unfortunately, this does not estimate the Δ_i^r, Δ_i^α values for all radars. To resolve this problem, the additional information about the functions $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$ is now needed.

II. Parametric estimation algorithm for determination of systematic errors

The first algorithm for determination of the systematic errors was elaborated on the basis of the parametric estimation technique. This algorithm processes separately each aircraft trajectory and gives its individual estimate for the systematic errors (the first stage of algorithm). The final field of systematic error shifts $s_i(x)$ is evaluated by the averaging procedure using the individual estimates that have been got previously (the second stage of algorithm). It is supposed that the number of aircrafts and the durations of their tracks are sufficient for the adequate averaging. Additionally, it is important that the spatial coverage of the aircraft tracks must be sufficiently wide for the confidence of the inferences about the structure of the functions $s_i(x)$. Also, we suppose that the instants of measurements are exactly known.

The algorithm can work with observations from primary surveillance radars (PSR) without any information about altitude of the aircraft observed. This important peculiarity takes place due to the simplicity of this two-stage algorithm.

A. Estimation of systematic errors for one aircraft

For evaluation of the individual aircraft trajectories, it is important that all radar tracks to be preliminary cleared from outliers. So, the first procedure in this estimation algorithm consists of the preliminary removing of defective measurements.

Only such parts of aircraft trajectories are taken into further evaluations that have been observed by two or more radars simultaneously. If a part of a trajectory has been observed by only one radar, then this part would be removed from the evaluation process.

We introduce the notion of an auxiliary *reconstructed track* as a linear-piecewise three-dimensional curve that approximates the true aircraft motion. Nodes of the reconstructed track correspond to the preliminary fixed time grid.

In the case when the measurement includes information on the altitude, this measurement is a point in the three-dimensional space. Coordinates of such a point depend on the radar coordinates, the slant range, azimuth, altitude, and, additionally, on expected errors in the azimuth and range. If the measurement does not contain information on the altitude (this is the case of the primary radar or when the measurement of the secondary radar has no valid value of the altitude), then we have an uncertainty in the form of some arc of a circle in the three-dimensional space. The location of this arc depends on the radar coordinates, slant range, the measured azimuth, and on expected errors in the range and azimuth.

For each measurement (with a number j of the radar i), we can calculate its residual $\delta_{i,j}$, i.e., the squared distance between the measurement uncertainty (this can be a point or an arc) and the point on the reconstructed track corresponding to the measurement instant. The residual depends on data of the measurement (the azimuth, slant range, and, possibly, the altitude), on the radar coordinates and coordinates (x_k, y_k, z_k, x_{k+1}, y_{k+1}, z_{k+1}) of two points of the reconstructed track, and, additionally, on expected systematic errors (Δ_i^α and Δ_i^r; here, it is assumed that the systematic errors do not depend on the measured azimuth and slant range) of the radar considered:

$$\delta_{i,j} = \delta_{i,j}\left(\Delta_i^\alpha, \Delta_i^r, x_k, y_k, z_k, x_{k+1}, y_{k+1}, z_{k+1}\right).$$

This formula shows the dependence only on those arguments that will vary.

By summation of all the residuals related to this aircraft, we obtain the aircraft *summary* residual D:

$$D = \sum_i \sum_j \delta_{i,j}\,.$$

It depends on the coordinates of all points of the reconstructed track, the data of all measurements of this aircraft, on the coordinates of all radars observed this craft, and on systematic errors of all these radars. Coordinates of points of the reconstructed track and expected systematic errors of the radars can be regarded as independent variables that can be varied:

$$D = D\left(\Delta_1^\alpha, \Delta_2^\alpha, \ldots, \Delta_1^r, \Delta_2^r, \ldots, x_1, y_1, z_1, x_2, y_2, z_2, \ldots\right).$$

So, we reduce the problem to the minimization of a function of many variables:

$$D\left(\Delta_1^\alpha, \Delta_2^\alpha, \ldots, \Delta_1^r, \Delta_2^r, \ldots, x_1, y_1, z_1, x_2, y_2, z_2, \ldots\right) \to \min.$$

As a result of solving this problem, we obtain values for the point coordinates of reconstructed track. Simultaneously, we implement the reconstruction of some "mean" trajectory of the aircraft and find the expected systematic errors that are consistent in the best way with the problem parameters (the data of measurements and coordinates of the radars).

For solving the minimization problem of the function of many variables, the Hooke–Jeeves algorithm [1] is used with some small modifications.

On the initial stage of investigations, it was assumed that the systematic error in range is absent, and the systematic error in azimuth for each radar is a value that does not depend on the measured azimuth and slant range. But the application of such a model for the processing of real data gives often unsatisfactory results. Namely, the dispersion of the systematic error found for various aircrafts may be very large. Simultaneously, it was noted that there is a correlation of the systematic error with the mean azimuth of the radar track. This allowed one to make a hypothesis about dependency of the systematic error in azimuth on the measured azimuth. The linear-piecewise and trigonometric dependencies of the azimuth systematic errors vs. the measured azimuth were implemented in the software. In the processing of real data, the trigonometric dependency is less convenient than the linear-piecewise one.

Moreover, the relative systematic error in the range has been introduced into the software. This error is taken to be constant and individual for each radar. An example of results of the algorithm work is presented in Fig. 3 for one aircraft.

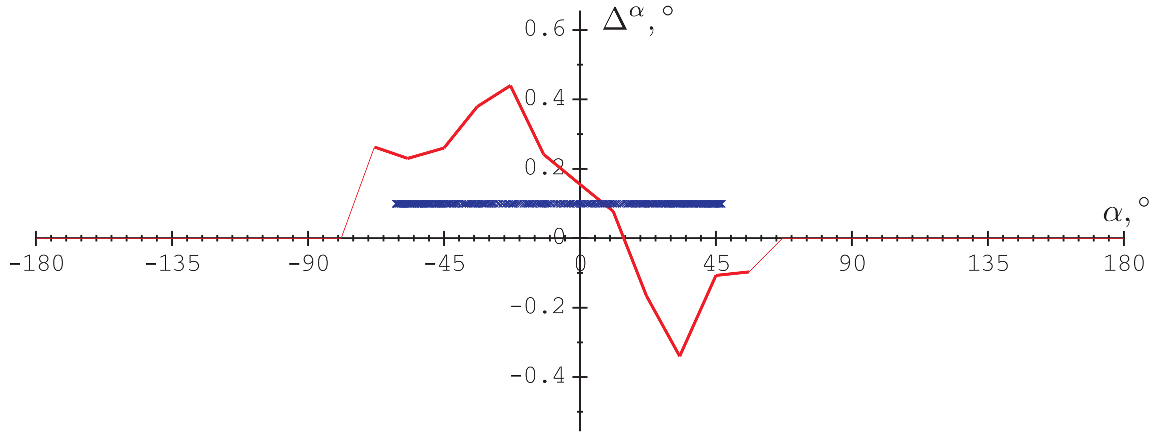


Figure 3. Example of reconstruction of the systematic error in azimuth for one aircraft; the red linear-piecewise line is the dependence of the systematic error in azimuth vs. the measured azimuth; the horizontal coordinates of blue markers are the measured azimuths, and the vertical coordinate of markers is the mean value of the systematic error

B. Statistical procession of tracks for many aircrafts

By many reasons, the results of the processing of the tracks for only one aircraft can not be used for other aircrafts. In particular, the region of azimuth measurements for one aircraft may not coincide with such a region of other craft. But, it is possible to construct the whole average picture of the radar systematic errors in the observation zone. It can be done on the basis of the processing of the large data on aircraft motions (in this zone) during a sufficiently large time period (from several hours to the whole day-time). Such results will not contain drawbacks of the processing of the only one aircraft data. An example of the average systematic error dependency in azimuth vs. the measured azimuth is shown in Fig. 4.

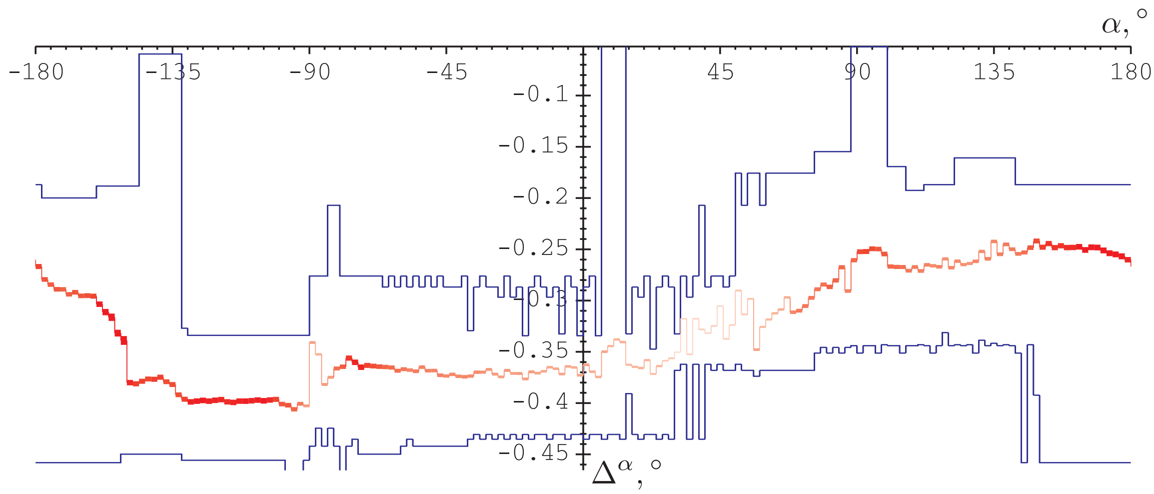


Figure 4. Example of the systematic error dependency in the azimuth measurements for one radar based on the results of the processing of measurements of many aircrafts by many radars in the Moscow Zone (Russia); the red central line is the averaged estimate of the systematic error; the marginal lines correspond to the minimum and maximum of the individual estimates for the systematic errors

The result has been obtained on the basis of the processing of a large data array. Measured azimuths and slant ranges are different for various aircrafts and radars. So, under statistical procession, we can obtain the systematic error dependency in azimuth and the systematic error dependency in range vs. both on the azimuth and range. These dependencies can be presented in the form of a vector field, and they can be used for the correction of the radar current measurements both in the range and azimuth (see Fig. 5).

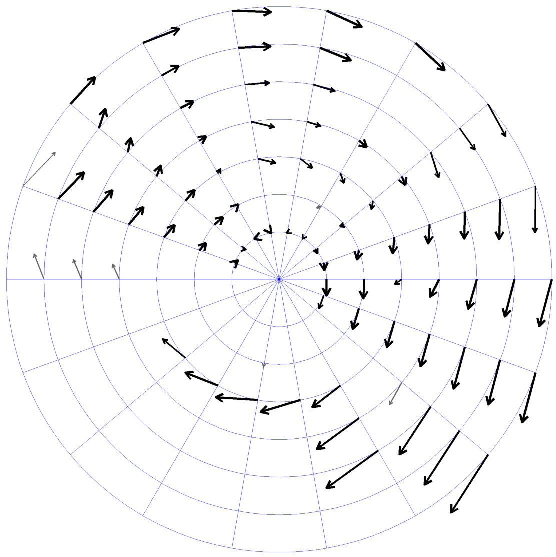
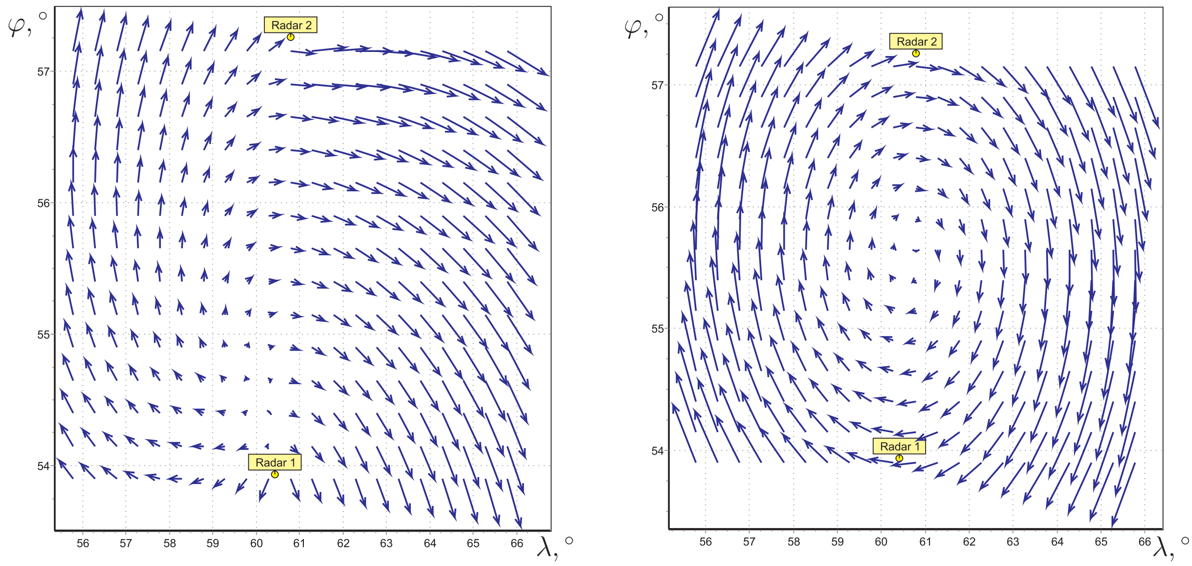


Figure 5. The systematic error dependency for one radar is presented in the form of a vector field of the measurement shifts; this dependency is obtained by the processing of data from many aircrafts in the Novosibirsk Zone (Russia)

III. Geometric approach to the determination of radar systematic errors

In this section, we assume that the radars (that participate in observation of an ATC aerial zone) have unknown but constant systematic errors in azimuth and range. The range error is assumed to be additive and independent of the measured range.

If to consider (under the mentioned assumptions) two radars that have overlapping observation zones, then the mutual effect of errors will give a vector field of the geometric discrepancy of measurements for these radars. We call such a field the field of *relative* errors of these radars. Figure 6 shows possible model versions of such fields with mutual geometric errors of two radars. Here, in projections into the plane of the geographic coordinates, the vectors of resultant relative discrepancy of measurements are given on some uniform grid in the latitude and longitude. The distance between these two radars is about 370 km; for apparency, the size of vector shifts w.r.t. their actual values is enlarged by 50 times. The difference vector field for the pair of radars with its exact positions (in the assumption that the positions are separated) defines by a unique way the values of systematic errors in azimuth and range for both radars. Correction shifts with the inverse values of systematic errors compensate the actual discrepancy of the radar measurements in the considered zone of geographic coordinates. Such a problem is easily solved numerically by direct finding of the constant systematic errors for both radars in azimuth and range.



(a) Radar 1: $\Delta_1^r = 0\,\text{m}$, $\Delta_1^\alpha = 0.2\,°$;
Radar 2: $\Delta_2^r = 0\,\text{m}$, $\Delta_2^\alpha = -0.2\,°$.

(b) Radar 1: $\Delta_1^r = 500\,\text{m}$, $\Delta_1^\alpha = 0.2\,°$;
Radar 2: $\Delta_2^r = -500\,\text{m}$, $\Delta_2^\alpha = 0\,°$.

Figure 6. Model versions of two vector fields of pairwise radar geometric errors

The procedure for calculation of these corrections in azimuth and range could be easily expanded to the case of three and more radars if corresponding pairwise estimations of the geometric discrepancy are known. Under this, we minimize the sum of squared differences between the model discrepancies (that have been obtained by necessary choice of constant systematic errors in azimuth and range) and discrepancies calculated on the basis of the input data (the actual discrepancies). For aligning tracks from a large number of radars, one uniform grid of the geographic coordinates is used.

In the described scheme of calculating the estimation of the radar constant systematic errors in azimuth and range, the most difficult operation is obtaining the input information about the geographic discrepancy of measurements from a pair of two radars. In practice, if to take into consideration a small geographic region, we have (as a rule) measurements of different accuracy and, moreover, coming from the corresponding radars with different frequency at different instants. The period of coming the measurements from surveillance radars can achieve 20 s.

For a small region π with a center at the geographic coordinates (φ, λ), the systematic errors Δ_i^r and Δ_i^α for the radar with number i approximately correspond to a constant shift vector $s_i(\varphi, \lambda, \Delta_i^r, \Delta_i^\alpha)$ in the local horizontal plane. In this plane, the vector has the following description:

$$s_i(\varphi, \lambda, \Delta_i^r, \Delta_i^\alpha) = \begin{bmatrix} \cos\alpha_i & -\sin\alpha_i \\ \sin\alpha_i & \cos\alpha_i \end{bmatrix} \begin{bmatrix} \Delta_i^r \\ r_i \Delta_i^\alpha \end{bmatrix},$$

where r_i, α_i are the average range and azimuth from the radar with number i to the center of the small region π. Denote a difference vector between two shift vectors for radars i and j as

$$d_{ij}(\varphi, \lambda, \Delta_i^r, \Delta_i^\alpha, \Delta_j^r, \Delta_j^\alpha) = s_i(\varphi, \lambda, \Delta_i^r, \Delta_i^\alpha) - s_j(\varphi, \lambda, \Delta_j^r, \Delta_j^\alpha).$$

From real data, we can find the difference shift vector $\widetilde{d}_{ij}(\varphi, \lambda)$ that characterizes the divergence between measurements of radars i and j in the small region π with center (φ, λ). If these difference shifts are determined for all regions $\{\pi_n\}$, we can estimate constant systematic errors of all radars by minimizing the following criterion:

$$\sum_{i<j} \sum_{\{\pi_n\}} \|\widetilde{d}_{ij}(\varphi_n, \lambda_n) - d_{ij}(\varphi_n, \lambda_n, \Delta_i^r, \Delta_i^\alpha, \Delta_j^r, \Delta_j^\alpha)\|^2 \xrightarrow[\{\Delta_i^r, \Delta_i^\alpha, \Delta_j^r, \Delta_j^\alpha\}]{} \min .$$

For determination of desirable difference vectors $\widetilde{d}_{ij}(\varphi, \lambda)$ for the radar pairs, it is possible to use all tracks $\mathcal{A}$ of the aircrafts that were flying in the considered region π for some time interval. Each of two fragments of these tracks is regarded as a geometric figure (composed of a collection of piecewise-linear segments) in the projection onto the local horizon plane. The vertical coordinates of various radar tracks from one aircraft have close values because the altitude is usually observed by the aircraft itself. The difference vector of the shift is calculated by superposition of such figures with minimization of the mutual summary distance $R_{ij}(\varphi, \lambda)$ between the piecewise-linear segments of the tracks (Fig. 7). The summary distance $R_{ij}(\varphi, \lambda)$ consists of the "projections" $\delta(z_i(t_k))$ of the measurements of radar i onto the polyline with vertices at the measurements of radar j and of the "projections" $\delta(z_j(t_l))$ (which can be made in a vice-versa way):

$$R_{ij}(\varphi, \lambda) = \sum_{A \in \mathcal{A},\ \{t_k^i\}} \|\delta(z_i(A, t_k^i))\|^2 + \sum_{A \in \mathcal{A},\ \{t_l^j\}} \|\delta(z_j(A, t_l^j))\|^2,$$

where t_k^i, t_l^j are the instants of measurements of the radars i and j.

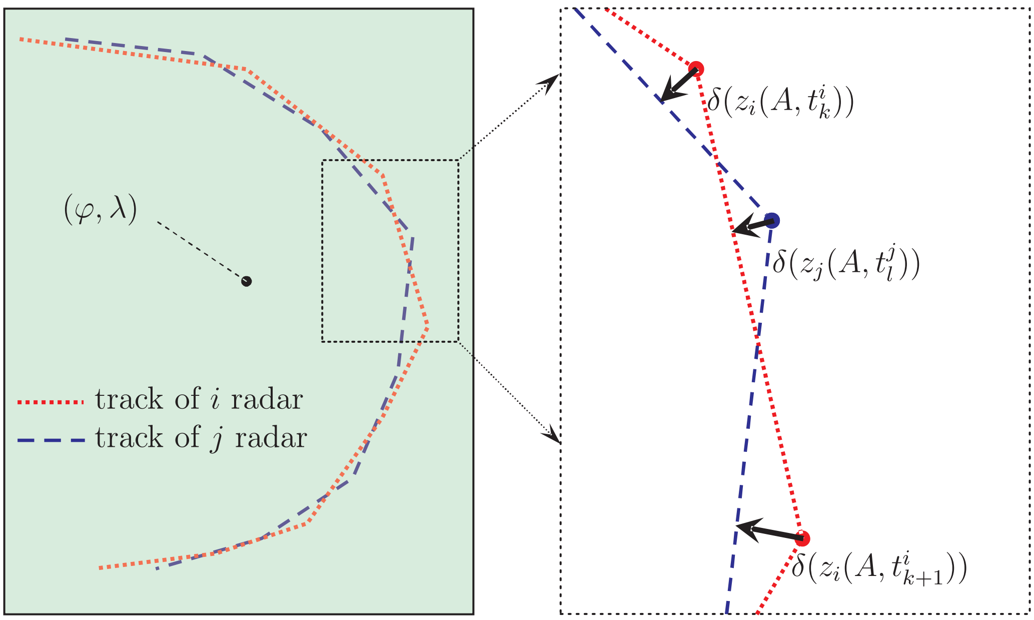


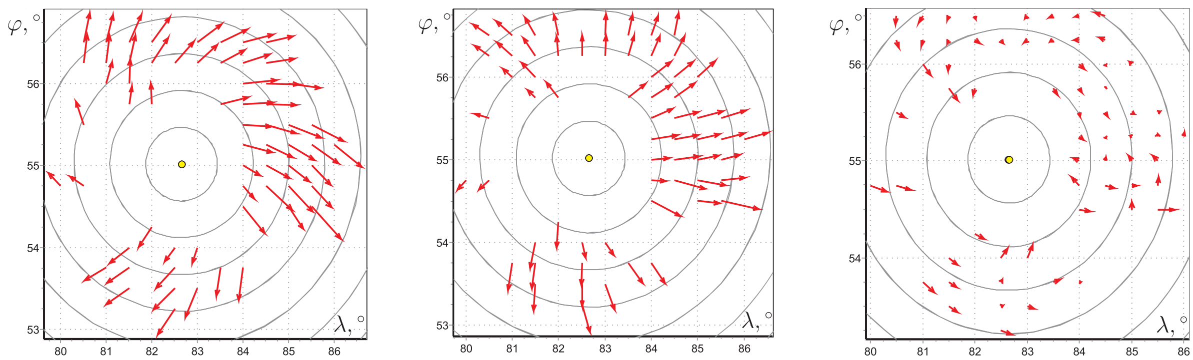
Figure 7. Calculation of the difference vector by superposition of the track figures from two radars

Initially, this method was applied in order to calculate the discrepancy in time of the data of radars. The geometric superposition of tracks allows one to eliminate the systematic errors in azimuth and range in calculating estimations of the mistiming. On the other hand, the time errors are eliminated during the calculation of the difference vectors of the shift, since the corresponding instants of the radar measurements are not used for superposition of tracks.

In some cases, the superposition procedure gives a high level of errors (even if we have a large number of measurements); for example, if trajectories of all the aircrafts are the parallel straight lines. Therefore, we need the presence of the segments of motion with different directions in a small geometric region. Also, the tracks should not have the form of a collection of concentric arcs.

To test the algorithm of this section, special programs were elaborated. They allow one to automatize the described computational scheme, including the choice of geometric regions for correct calculation of the discrepancy vectors for measurements of the radar pairs. Tests of the software show the consistent results, especially, for zones with high-density traffic. In such cases, the difference vector field of the radar discrepancies becomes the most representative.

For illustration, let us give some computational results for the Novosibirsk ATC zone. Using the data of continuous (during 8 hours) observation, the radar measurement discrepancy is calculated on some grid of the geographic coordinates. Further, the desirable corrections in azimuth and range have been calculated for each radar by using the procedure of optimization (described above). For one pair of radars with close positions, Fig. 8 represents the initial corrections (a), the residual shifts (b) after taking into account corrections only in azimuth, and the residual shifts (c) after taking into account corrections both in azimuth and range.



(a) the initial shifts; (b) after taking into account corrections only in azimuth; (c) after taking into account corrections both in azimuth and range.

Figure 8. Results of calculations for a pair of radars in the Novosibirsk ATC zone

IV. Non-parametric estimation algorithm for determination of systematic errors

Up to date, the main approach for determination of the radar systematic errors Δ_i^r and Δ_i^α consists of the parametric estimation [2,3]. In this approach, the structure (e.g., the model) of the functions $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$ is preliminary prescribed.

For example, it can be assumed that the systematic errors have the following type: $\Delta_i^r(x) \equiv 0$, $\Delta_i^\alpha(x) \equiv \Delta_i^{\alpha *} = \mathrm{const}$. In such a case, an algorithm should be constructed for determination of the unknown constants $\Delta_i^{\alpha *}$.

The assumptions about the structure can be more complicated but, in the parametric approach, they lead to formulas as follows:

$$\Delta_i^r(x) = \sum_{j=1}^{P^r(i)} a_i^{rj} f_{ij}^r(x), \qquad \Delta_i^\alpha(x) = \sum_{j=1}^{P^\alpha(i)} a_i^{\alpha j} f_{ij}^\alpha(x)$$

with preliminary defined functions $f_{ij}^r(x)$, $f_{ij}^\alpha(x)$ and their total quantities $P^r(i)$, $P^\alpha(i)$. In this approach, further evaluations are connected with determination of the unknown constants a_i^{rj}, $a_i^{\alpha j}$. The algorithms described in the previous sections belong to this parametric estimation class.

The functions $f_{ij}^r(x)$, $f_{ij}^\alpha(x)$ are usually chosen on the basis of the conceptions about the problem elaborated by engineers. The effectiveness of any parametric estimation algorithm strongly depends on this choice. But this choice can be bad.

In our case of the radar systematic errors, the model with the assumption

$$\Delta_i^r(x) \equiv 0, \quad \Delta_i^\alpha(x) \equiv \Delta_i^{\alpha *} = \text{const}$$

shows incompatibility with real data. In experiments, individual estimations of $\Delta_i^{\alpha *}$ for various aircrafts strongly differed from each other. Variation of the individual results was more than 10 times larger than the level that can be explained by influence of the random errors. In Fig. 9, the individual results for 7 trajectories are shown; each individual estimate is marked by the thick bar, and the "whiskers" (those that are drawn up and down) show the mean-square deviation level multiplied by 3.

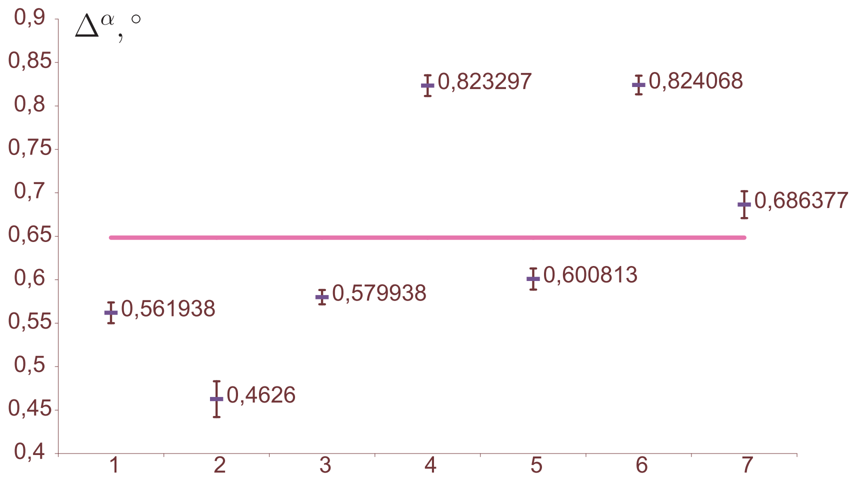


Figure 9. Divergence of results for individual trajectories

We alternatively attempt to estimate the systematic errors without using a fixed model.

All radar data are divided into individual aircraft trajectories, and each trajectory has several tracks from various radars. Let us make the preliminary smoothing of each track. This procedure could be made by various methods, for example, by the method of restoration of an aircraft trajectory [4]. In the sequel, only smoothed measurements will be considered. In the smoothed measurements, the level of random errors is sufficiently small. So, we can neglect the influence of these errors. This fact corresponds to setting the values of w_i^r, w_i^α, and w^h to be equal to zero in equalities (1), (2).

Consider a group $Z(t)$ of smoothed measurements for one aircraft from several radars at a certain instant t. In practice, real measurements from various radars come at different instants; but after the smoothing procedure, we can provide the measurements over the common set of instants.

Note that nothing about the structure of the functions $\Delta_i^r(x)$, $\Delta_i^\alpha(x)$ is known, besides that they have bounded values. So, the true aircraft position $x(t)$ at the instant t can be arbitrarily placed nearby the group $Z(t)$ of measurements. The aircraft position $x(t)$ is unknown, but its coordinates have common values for all radars. Hence, it is possible to construct a set of all versions of the systematic errors which are consistent with the given data (i.e., with a group of one-instant measurements) and with the fact that the vector $x(t)$ is the same for different radars. Let us call such a set the *uncertainty set* of the systematic errors for the given group of measurements and denote it by $F(Z(t))$. This set describes the dependency of possible systematic errors from each other for different radars. In the most demonstrative and simple way, this can be shown in the geocentric coordinate system for the geometric shifts conditioned by the systematic errors. Figure 10 shows some group of measurements and one of possible variants of the systematic error shifts.

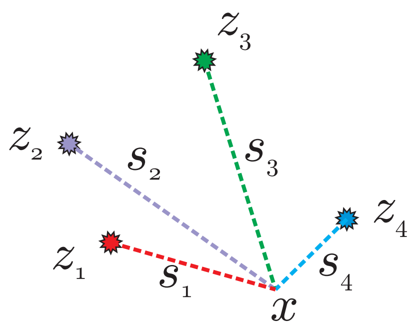


Figure 10. A variant of the systematic error shifts consistent with the group of measurements Z at one instant

During the analysis, it has been noted that the groups $Z(t)$ of measurements for various instants t look similarly to each other if they have close geographic positions. These groups change their forms rather slowly along any aircraft track. This allows one to introduce into consideration a new uncertainty set for the systematic errors, which is not connected with the measurements but with a certain spatial (or geographic) position. Such sets can be obtained by averaging all uncertainty sets $F(Z)$ over the small geometric region π around the given position x. Denote the uncertainty set at the given point x by $F(x)$.

From the analysis of $Z(t)$ groups, we can additionally conclude that a small change of the groups of measurements along tracks is likely connected with a small "slope" of the systematic error functions $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$ vs. the position x.

Given a grid $\{x_n\}$ of the spatial positions, it is possible to construct the set $F(x_j)$ at each point x_j by averaging all $F(Z(t))$ in the corresponding small region π_j. The collection of such sets for all x_j describes all possible variants of the radar systematic errors consistent with the given data (input measurements). We emphasize that the construction of the sets $F(x_j)$ does not use any model of the systematic errors and is not associated with any *a priori* assumptions on their internal structure. We use only the observation equation (1) that is based on the most simple and general information.

Note that the system described by equations (1) or (2) is not completely observed w.r.t. an aircraft unknown position $x(t)$ and the variables $\Delta_i^r(x(t))$, $\Delta_i^\alpha(x(t))$. This fact lies in the basis of the nature of the uncertainty sets $F(x_j)$. Instead of giving the estimations $\hat{\Delta}_i^r(x_j)$ and $\hat{\Delta}_i^\alpha(x_j)$ for the true values $\Delta_i^r(x_j)$ and $\Delta_i^\alpha(x_j)$ at the point x_j, we can only estimate some linear combinations of the components of the compound vector

$$f(x_j) = \begin{bmatrix} \Delta_1^r(x_j) \\ \Delta_1^\alpha(x_j) \\ \vdots \\ \Delta_m^r(x_j) \\ \Delta_m^\alpha(x_j) \end{bmatrix}.$$

The uncertainty set $F(x_j)$ at some position x_j is strongly connected with an estimate $\hat{h}(x_j)$ of $C(x_j)f(x_j)$. The matrix $C(x_j)$ depends on the position x_j and set of the observing radars at it. Our estimation minimizes the value

$$\mathbf{E}\left\{\|\hat{h}(x_j) - C(x_j)f(x_j)\|^2\right\}$$

in the class of linear estimators at every position x_j.

Using the constructed uncertainty sets $F(x_j)$, it becomes possible to build (by one or another way) an one-valued estimation of the systematic errors $\Delta_i^r(x_j)$ and $\Delta_i^\alpha(x_j)$ at the grid points $\{x_n\}$. Under this, if to choose values $\Delta_i^r(x_j)$ and $\Delta_i^\alpha(x_j)$ from the set $F(x_j)$, then we can be assured in their complete consistency with the given measurements. At the points different of the grid ones, the systematic errors can be calculated by interpolation.

The true one-valued functions of the systematic errors $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$ are unknown. But it is possible to accept some reasonable assumptions on such functions. As such an assumption, we can take the demand of vicinity of the functions $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$ to some constants at the most points of the grid $\{x_n\}$. Other assumptions having engineering sense can also be made.

The authors think that the most natural assumption is the demand of a small slope of the functions $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$ w.r.t. the spatial position x. Such a demand can be formalized by means of a functional

$$J(f(\cdot)) = \sum_{x_i, x_j \in \chi} \frac{\|f(x_i) - f(x_j)\|^2}{\|x_i - x_j\|^2}$$

having the sense of the average (over the points $\{x_n\}$) square of the Lipschitz constant for the function under investigation (the set χ consists of pairs of the nearest points x_i, x_j from the set of positions $\{x_n\}$). The functions $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$ close to the true ones should give small values of such a functional. The functions minimizing the corresponding functionals can be regarded as "candidates" for the true systematic errors.

Now, we elaborate an algorithm that allows one to construct the functions $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$, which, on the one hand, provides the minimum of the functional and, on the other hand, the values $\Delta_i^r(x)$ and $\Delta_i^\alpha(x)$ belong to the uncertainty sets $F(x_j)$. In Fig. 11, we present a solution for the $\Delta^\alpha(x)$ systematic error of the "Kemerovo" radar. The solution is estimated by means of the algorithm of this section. Colors in the figure correspond to the value (in degrees) of the $\Delta^\alpha(x)$ azimuth systematic error. The location of the radar is marked by the cross. The small circles correspond to the grid $\{x_n\}$ of positions in the algorithm.

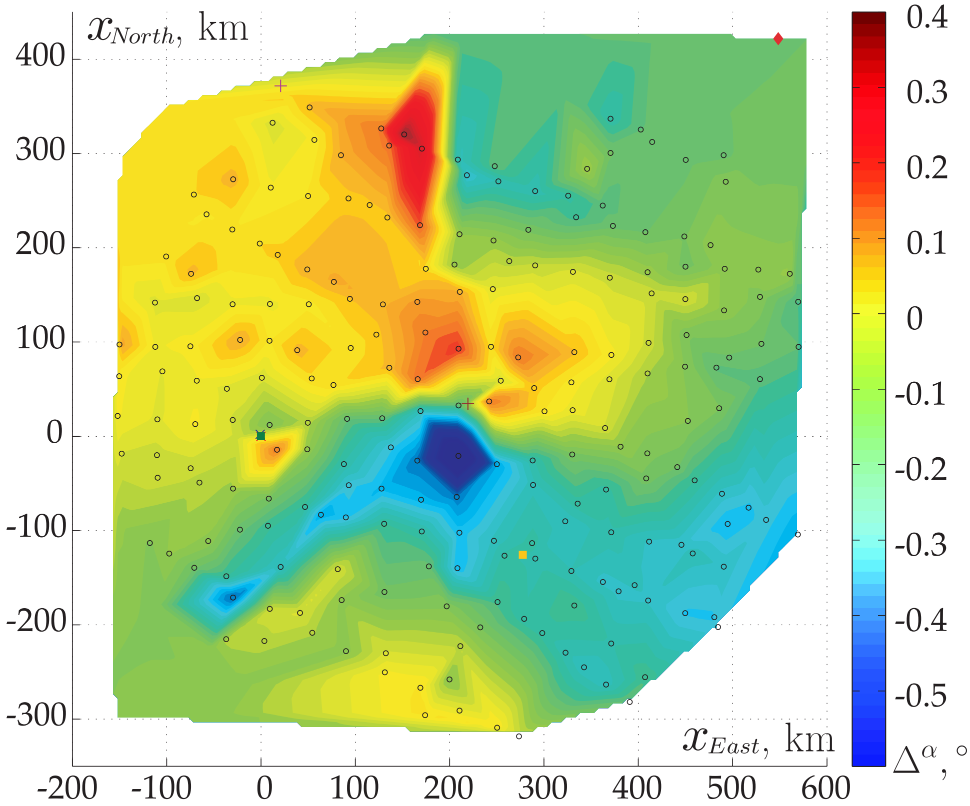


Figure 11. The solution for the azimuth systematic error $\Delta^{\alpha}(x)$ for "Kemerovo" radar

V. Conclusion

In the paper, three methods of determination of the radar systematic errors have been described. All these methods use only radar information without any additional sources (e.g., from ADS-B). The first algorithm focuses on parametric estimation and processes aircraft trajectories separately. The second algorithm does not use any information about measurement instants and is stable under time delays in information channels. The third algorithm does not use any prescribed structure of systematic errors (if they are considered as functions of an aircraft location). All these algorithms have been tested both on the simulated and real radar data.

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