28-30 MAY 2018

25th ANNIVERSARY SAINT PETERSBURG INTERNATIONAL CONFERENCE ON INTEGRATED NAVIGATION SYSTEMS

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RECOVERING AN AIRCRAFT TRAJECTORY BY USING THE DETECTION OF THE MOTION TYPE*

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***Abstract*—We describe the algorithm for recovering an aircraft trajectory that is based on the construction of a bundle of trajectories, which are the most possible versions of the real aircraft motion. The specific feature of the algorithm is the procedure for detecting the current motion type, which makes possible to improve the positional accuracy for determining the coordinates on the stages of steady motion. The results of the algorithm applied to model data are presented.**

Keywords—flight trajectory tracking; multi-hypothesis algorithm

We consider the task of on-line aircraft trajectory recover according to incoming radar measurements: after the arrival of the next radar measurement, the algorithm must immediately yield an estimate of the aircraft position. The main difficulty in the problem is that the object moves non-stationary, i.e., it performs maneuvers whose characteristics and duration are unknown to the observer. Here, long-term sections where the motion type is constant are possible, on which the aircraft trajectory is well approximated by one simple model. In such sections, it is important that the recovery algorithm should yield accuracy close to the accuracy of algorithms specially designed for this particular type of motion. In addition, for work in real conditions, the robustness to "outliers" in measurements is important. There is a large number of failures in real data that may lead to a "loss of the trajectory" by the recovery algorithm.

In spite of the existing solutions [1], [2], there constantly appear a lot of publications on various aspects of this problem; see, for example, [3]. They consider the trajectory processing for not only for the aircraft motion, but also for other objects as well; various mathematical methods are used, including those involving the use of multiple motion models (see, for example, [4]). The effectiveness of this approach lies primarily in the timely detection of the motion type (the localization of maneuvers).

In the present paper, one of the possible solutions of this problem is considered. The results of processing typical model data are presented.

I. General Algorithm Description

The basic structure of the algorithm is a set (bundle) of "most probable" aircraft trajectories, which is constructed taking into account the dynamics of the aircraft and possible outliers of measurements. The ends of the trajectories in the bundle are used to construct an estimate of the aircraft position. This estimate is issued as a result of the algorithm at the current time.

We assume that the aircraft moves in a horizontal plane according to the standard model of the simplest airplane motion [5], [6] (x and z are coordinates on the plane, φ is the path angle, and v is the speed):

$$\dot{x} = v\cos\varphi\,,\ \dot{z} = v\sin\varphi\,,\ \dot{\varphi} = u/v\,,\ \dot{v} = w\,.$$

In the case of constant tangential w and transversal u controls, these equations can be integrated analytically [7]. Each trajectory of the bundle corresponds to the dynamics and piecewise constant controls u and w. It is assumed that the duration of the sections of constancy cannot be less than a certain given constraint.

At the beginning of the algorithm work, the bundle startup procedure is executed with several first measurements. Then the main cycle is running where each iteration is connected with receiving a new measurement.

The track bundle is recalculated using measurements from the sliding time window of a fixed duration ending with the last measurement. The recalculation is started at each newly arrived measurement. A bundle of trajectories is formed with a view to support the maximum representativeness of various variants of motion.

For each trajectory of the bundle, the "accordance with measurements" criterion is calculated taking into account the distance between the trajectory and the measurements, as well as additional penalties.

Several criteria with different properties are used. The following properties are common to all criteria

- the smaller value of the criterion corresponds to the trajectory that is closer to the measurements;

- if the trajectory exactly passes through the measurements, the value of the criterion is zero.

Additionally, penalties are charged:

- for exiting the limitation on the maximum absolute value of the transversal and tangential controls;
- if the duration of a constant control section is less than the prescribed value;
- if the duration of two adjoining constant control sections is less than the prescribed value;
- if the value of the aircraft velocity module is small or large;
- if the motion type does not correspond to the type determined by the motion detector.

II. Basic Procedures of the Algorithm

A. Track extension and track trimming.

At this stage, the predicted position of all tracks is calculated at the time of the newly arrived measurement. The last section of constant controls is extended until the moment of the current measurement. On the other hand, the tracks are shortened in time from the old measurements, so that the total duration of the track does not exceed the preset window length.

B. New measurement branching.

Branching is a procedure in which, for each trajectory, possible variants of its extension are constructed with altered (with respect to the original trajectory) controls. There is a continuous "gluing" of the branch with the parent trajectory at an intermediate point. Choosing different branch points on the initial trajectory and different control values in the section after the branch, we obtain different variants of the trajectory. Only a few of all the possible options will be left in the bundle. The best values of the accordance criterion select the trajectories for remaining.

One of the variants of the branching is the trajectory that hits exactly the point of the last measurement. To construct this trajectory, we use the solution of an auxiliary problem of hitting a point described in [7]. Other options are also used: a branch with zero control and branches that hit random points near the last measurement. A branch with zero values of controls is intended to improve the approximation of measurements in areas where the aircraft finishes the maneuver and starts moving uniformly along straight line.

At the same stage, special trajectories are formed, namely, "OLS straight line" and "OLS circle", which are calculated without using any trajectory of the bundle as the parent path. The root-mean-square deviation of the constructed trajectory from the measurements is minimized. The "OLS straight line" assumes constant tangential acceleration and zero transversal acceleration. The "OLS circle" is constructed with zero tangential acceleration and constant transversal acceleration.

C. Preliminary bundle pruning.

At this stage, the trajectories that are poorly aligned with the available measurements and with physical limitations are deleted.

D. Selective optimization.

Optimization means the variation of the values of controls and switching times between the sections of constant control. A direct search method for finding the minimum of a multidimensional function is used. The optimization procedure applied to all the trajectories leads to poor results due to the "thinning" of the bundle and the loss of multi-hypotheses. Therefore, optimization is carried out only over a small number of trajectories with the best value of the criterion.

E. Calculation of the current position of the aircraft.

At each moment, when the measurement is arrived, the algorithm must produce an estimate of the aircraft position as an output. We use the averaging of the positions at this time for the trajectories of the bundle.

The estimation using the same criterion as in the basic procedures does not always yield good results. In the described version, the evaluation of the current position is generated using weights derived from other criteria of quality.

Not all available trajectories of the bundle are involved in the estimation, but only those for which the value of the main criterion of accordance is small. For each trajectory, its weight is calculated. Depending on the detected type of current motion, the weight of the "OLS" trajectories can be forcibly increased.

F. Grouping and pruning.

The task of this procedure is to reduce the number of tracks in the bundle while maintaining the representativeness of different hypotheses about the aircraft motion. A pair of trajectories with a minimum distance is determined in the matrix of mutual distances between tracks of the bundle and the trajectory of this pair with the worst criterion is removed from the bundle. Then we again look for a pair of trajectories with the minimum distance, etc. The procedure continues until the number of trajectories is less than the prescribed number.

III. Motion Type Detecting

To detect the motion type, the following scheme has been developed. The main algorithm forms the evaluation of the tangential and transversal accelerations. Each of them is analyzed separately by a special algorithm, which we will later call the "detector". The purpose of the detector is to discover that the input signal is close to constant, or vice versa, to discover its sudden change after a period of constancy.

Consider the detector in more detail. Let a certain function $u(t)$ be measured at discrete time instants t_i; its value at the time t_i will be denoted u_i. We denote by t_f the current time for which the analysis is carried out; the index f for other values will mean that they correspond to this time.

The detector has two modes: the mode for searching the constancy of $u(t)$ and the mode for searching the end of the $u(t)$ constancy section.

A. Search mode for a section of constancy

The purpose of the work in this mode is to determine that the function $u(t)$ is close to the constant one on the interval starting from some instant t_c to the current time t_f. The work proceeds based on constructing the average values over the system of interval from the current time t_f to a certain depth. Namely, $\bar{u}_i = \sum_{j=i}^{f} u_j \big/ (f-i+1)$ is the average value for the interval $[t_i, t_f]$ from the instant t_i to the instant t_f. Next, the system of nested intervals, $[t_f]$, $[t_{f-1}, t_f]$, …, $[t_{f-N}, t_f]$, to the maximum depth N (the length of the window in the main algorithm) is considered. We denote by $[\cdot]$ the "indicator" operation, which returns 1 if the expression within the brackets is true, and 0 if false.

The algorithm provides for the analysis of the quantity $\xi_i = \sum_{j=i}^{f} \left[\left| u_j - \bar{u}_j \right| \le \delta_u \right]$. This quantity has the following meaning: how many times, from the instant t_i to the instant t_f, the value of the function u_i has been good "predicted" by the mean value over the moments to the right from t_j (the criterion of "good prediction" is the location in the corridor of width δ_u).

We consider a sequence of quantities ξ_i at some given depth n from the current time: ξ_f, ξ_{f-1}, …, ξ_{f-n} ($n < N$). If $\xi_{f-n} > m$, where m is a predetermined integer, the decision is made that the observed function $u(t)$ has a constant section terminating at the current point t_f. We estimate the start time t_c of a constant section based on the condition $c = \max\{i : \xi_i = m\}$.

The work of the algorithm is completely determined by the constants: δ_u is the width of the corridor; n is the depth of view backwards for the formation of ξ_i; and m is the number of "good predictions" sufficient to make a decision. The algorithm currently has no full theoretical justification, but it showed good results in tests.

B. Finding the end of the constancy section

Real aircraft trajectories consist of sections where controls are constant. Each of such intervals is limited. If the detector has determined that the function $u(t)$ is in the constancy section, an algorithm that monitors its possible termination is started. The algorithm is based on comparing the current value u_f with the mean value $\hat{u}_f$ calculated using the time points to the left from t_f to the depth $c' = \max\{c, f-N\}$:

$$\hat{u}_f = \frac{1}{f-c'} \sum_{j=c'}^{f-1} u_j \, .$$

Here, c is the index of the time instant t_c defined as the beginning of a constancy section.

Using the value $\hat{u}_f$, we calculate the attribute of "exit from the corridor": $\zeta_f = \sum_{j=i}^{f} \left[\left| u_f - \hat{u}_f \right| > \delta_u \right]$. If the attribute is not fulfilled, that is, $\zeta_f = 0$, then we conclude that the constancy section continues at the time t_f.

If $\zeta_f = 1$, this can be for different reasons:

- the section of constancy of the function $u(t)$ actually ended;
- there was an outlier in the data and the value u_f contains a large error.

In order to exclude the effect of outliers, the following rule was adopted. In the case $\zeta_f = 1$, the average value $\hat{u}$ of the function $u(t)$ is stored to the left of t_f; i.e., we perform the assignment $\hat{u} = \hat{u}_f$.

Then, with the subsequent values u_{f+1}, u_{f+2}, … of $u(t)$ delivered, they are compared not with $\hat{u}_{f+1}$, $\hat{u}_{f+2}$, …, respectively, but with the "frozen" value $\hat{u}$. This is done in order to exclude the effect of outliers or abrupt changes on $\hat{u}$, which should respond to the value of the function in the last constancy section. The rule $\zeta_i(\hat{u}) = \left[\left| u_i - \hat{u} \right| > \delta_u \right]$ calculates the attributes $\zeta_{f+1}(\hat{u})$, $\zeta_{f+2}(\hat{u})$, …, $\zeta_{f+k}(\hat{u})$ to the depth k. The calculations are terminated if $\zeta_{f+i}(\hat{u}) = 0$; so, the function has returned to its normal value. If all the consecutive values $\zeta_{f+1}(\hat{u})$, $\zeta_{f+2}(\hat{u})$, …, $\zeta_{f+k}(\hat{u})$ are equal to 1, the decision is made that the constancy section has ended. The time t_f is memorized.

IV. Modeling Results

An ideal model trajectory is formed that consists of steady motion sections and transition sections for which the maximum tracking errors are defined in the standards [8]. For this trajectory, 100 model tracks of measurements with a mean square deviation of 70 m were generated. For each of them, the trajectory was restored using the described algorithm. The graphs of the root-mean-square deviation of the restored positions from the true motion are constructed. The time plot of the tangential deviation is depicted in Fig. 1 (thick solid line). Also, a similar graph for the Interacting Multiple Model (IMM) algorithm [1], [2] is given (dashed line). The thin solid line shows the graph of root-mean-square deviation of measurements.

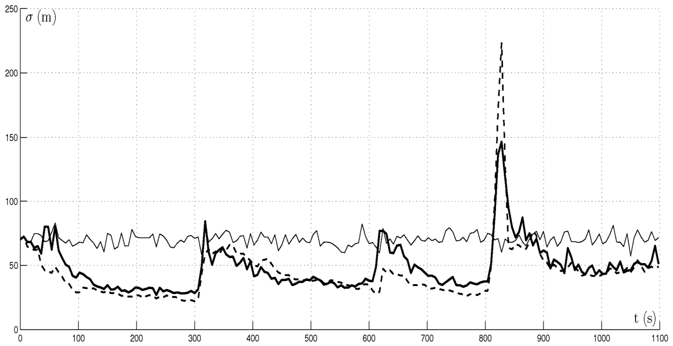

Fig. 1. Graph of the root-mean-square longitudinal deviation σ as a function of time. The thick solid line shows the results by the described algorithm. The dashed line is the results by the Interacting Multiple Model (IMM) method. The thin solid line is the track of measurements

In the case of processing trajectories with outliers (rare large deviations that do not be drawn from regular distribution), the advantage of the proposed algorithm becomes more evident.

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