Adaptive Control in Three-Dimensional Linear Systems with Dynamical Disturbance of Unknown Level[1]

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Abstract: *The construction of adaptive control in two-person linear differential games with a three-dimensional terminal set is considered. The level of constraint on the second player's control is not known in advance. The algorithm is based on the construction of the system of maximal stable bridges for conventional differential games with prescribed constraints on the controls of both players.*

Based on methods of the differential game theory [1], a way for the construction of adaptive control was suggested in [2] for problems with an unknown level of dynamical disturbance. The method is applied to systems with linear dynamics, fixed terminal time, and convex terminal set. It is assumed that the useful control is scalar and constrained in modulus. The aim of control is in getting the phase vector onto the terminal set at the terminal instant and as close to its "center" as possible. The dynamical disturbance is also assumed to be scalar and constrained, but the level of this constraint is not known in advance.

The adaptive control is constructed based on the family of stable bridges [1], and each bridge represents a tube in the space *time*×*phase vector* and corresponds to some value of a numerical parameter. The family of tubes is ordered by inclusion with increasing the parameter. An antagonistic differential game with geometrical constraints on the first and second players' controls and its own terminal set corresponds to each value of the parameter. In the original game, the first player is identified with the useful control, and the second player is identified with the disturbance. The property of stability allows the first player to hold the system motion inside each tube under the corresponding level of the disturbance.

Let a disturbance not exceeding some level act onto the system. Then, if the adaptive control is used, the controlled system moves along the considered family of bridges up to the

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bridge that corresponds to this level of the disturbance. After this, the motion does not fall outside the boundary of this bridge, i.e., it does not go over to the larger bridges. Thus, the system automatically adjusts the level of the useful control to an unknown level of the disturbance.

In realization of the control method suggested in [2], the main difficulty is in the ability of constructing the bridges and forming the system of these bridges. In [2], an algorithm is described that allows us to do this for the case when the terminal set is completely defined only by two components of the multidimensional phase vector of the original linear system. In this case, the passage is possible to the equivalent constructions in the space *time*×*two-dimensional phase variable.* Therefore, the phase space in these constructions is two-dimensional.

This presentation is devoted to the algorithms of constructing a system of embedded stable bridges and the corresponding adaptive control for the case when the terminal set in the original problem is defined by three components of the phase vector. The principal complication is concerned with the constructions in the three-dimensional phase space of equivalent variables.

For each stable bridge, any its *t*-section (here, *t* is a time instant) represents itself a three-dimensional convex body that is approximated by a convex polyhedron. The bridge is constructed by means of the backward procedure on the given grid of the time instants $t_0, \ldots, t_n$. The rule of the passage from the section at the instant t_{i-1} to the section at the instant t_i, where $t_i < t_{i-1}$, is connected with operations of computation of the algebraic sum of the polyhedrons and their intersection. The main ideas of the computational algorithm for such operations are taken from [3].

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