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Politechniki Wrocławskiej

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**13<sup>th</sup> International Symposium  
on Dynamic Games and Applications**

Wrocław 2008



## 13<sup>th</sup> International Symposium on Dynamic Games and Applications

Edited by  
**Arik A. Melikyan, Andrzej S. Nowak,  
Krzysztof J. Szajowski**

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Wybrzeże Wyspiańskiego 27, 50-370 Wrocław  
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# 13<sup>th</sup> International Symposium on Dynamic Games and Applications

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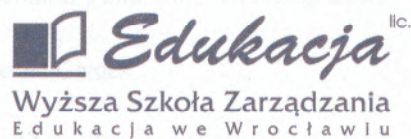
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## Maximal stable bridges in linear differential games of the third order

Alexey Ivanov

Andrey Ushakov  
Institute of Mathematics and Mechanics  
16, S.Kovalevskaja street  
620219, Ekaterinburg GSP-384  
Russia  
e-mail: iagsoft@imm.uran.ru, beerzone@olympus.ru

### Abstract

In the theory of antagonistic differential games, a very important problem is constructing the collection of all initial positions, wherefrom the first player guarantees guiding the phase vector to the given terminal set at the terminal instant despite of the action of the second player. After the work [1], such a set is called the maximal stable bridge.

In this work, an algorithm is considered for constructing maximal stable bridges for systems with a linear dynamics, scalar players' controls and a convex compact terminal set, which is defined by some three components of the phase vector. The latter allows to pass to an equivalent differential game with the phase variable of the third order. With that, the section of the maximal stable bridge (if the section is non-empty) is a convex three-dimensional set at any time instant. During numerical constructing, these sets are approximated by convex polyhedra.

The suggested algorithm for backward constructing time sections is based on a procedure for computing convex hull of a piecewise-linear positively homogeneous function defined in a three-dimensional space. The peculiarity of the procedure is that it uses the known information about places, where the local convexity can be violated. The algorithm follows the scheme described in [2].

Results of constructing maximal stable bridges for some examples are given.

### Bibliography

- [1] N.N. Krasovskii and A.I. Subbotin. *Game-Theoretical Control Problems*. Springer-Verlag, New York, 1988.
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