IDENTIFICATION OF SYSTEMATIC ERRORS IN RADAR AZIMUTH MEASUREMENTS ON THE BASIS OF OBSERVATION OF AIRCRAFT MOTION *

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Abstract

Keywords: radar, identification of parameters, inverse problem, reconstruction of a motion trajectory, numerical simulation

A problem of identification of systematic errors in azimuth measurements by several radars is considered; observation of an aircraft motion is implemented on a certain time interval. The radar output data are presented by samples of measurements of the slant range and azimuth. Additionally to the computation of errors in azimuth, the trajectory of the aircraft is reconstructed in simplified form of a polygonal line in the three–dimensional space. The algorithms elaborated were tested on model and real radar data.

Radar systematic errors in azimuth lead to the spatial shift of the track of the aircraft under observation. If the same aircraft motion is observed by several radars, then there is possibility to calculate their systematic errors in azimuth. It is crucial that the considerable systematic errors are concerned only with the azimuth measuring, but measurements of the slant range do not contain large systematic errors.

The work deals with solving a problem of identification of systematic errors in azimuth measurements obtained from several radars. It is assumed that each of radars measures the slant range and azimuth with some error. Measurements are made in a local coordinate system of each radar. Measurements from different radars come with own time steps.

For large sizes of the air traffic control zones (the typical size is of 150 km or larger), application of the "flat–earth model" to computations gives large errors, so the "spherical–earth model" is used. The identification algorithm elaborated earlier and using the flat–earth model is described in [1].

For solving the identification problem, the notion of the "reconstructed track" is introduced. The reconstructed track is a polygonal line in the three–dimensional space. To each vertex of this line, a time instant is assigned. Coordinates of vertices can be regarded as independent variables. For fixed coordinates and any time instant, a point of the reconstructed track is put into correspondence (it is assumed that the time between the vertices of the line changes uniformly).

The slant range and azimuth of the radar measurement give rise to uncertainty of a position of the observed aircraft in the form of a circle in the three–dimensional space, and the circle lies in the plane perpendicular to the local horizon at the point of placing the corresponding radar.

The distance ρ (between the point $\vec{a}$ of the reconstructed track and the circle with the center at the point $\vec{c}$, the radius r, and the normal vector $\vec{n}$) is calculated by the formula

$$\rho = \sqrt{\left|\vec{l}\right|^2 + r^2 - 2r\left|\vec{n} \times \vec{l}\right|}, \text{ where } \vec{l} = \vec{a} - \vec{c}.$$

A non–decreasing function of the distance between the circle and the point of the reconstructed track (the point corresponds to the time instant of the radar measurement) is called the residual of this radar measurement. Simulation has showed that the quadratic distance function, which is non–decreasing for nonnegative values of the argument, is the most suitable to be taken as the residual.

Construct now a function whose arguments are the coordinates of the vertices of the reconstructed track and the systematic errors in azimuth of all radars observing the same aircraft. Using the systematic errors, let us

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make corrections in values of the azimuths for all radar measurements. After this, calculate and summarize up the residuals of all measurements. The sum is regarded as the function value. Then, the value of the argument providing the minimum of the function will be comprised of the apices of the reconstructed track that approximates the real trajectory of the aircraft and of the values of the systematic azimuth errors that provide the best match of the trajectories from different radars.

In this way, the problem of finding the systematic errors in azimuth can be formulated as a problem of determining the minimum point for a function of several variables.

For solving this minimization problem, we used the Hooke and Jeeves algorithm [2] that gives good results in the case of many variables. In the problem under consideration, the argument of the minimized function has dimension about 50–600 depending on the initial data and the solution parameters. To increase the chance of achieving the global minimum, the algorithm in [3] was completed with elements of the random search (i.e., the Monte Carlo method).

If in the radar data there is additional information on the aircraft altitude, the algorithm can take into account this information. In this case, the measurement residual is a non–decreasing function of the distance between a point of the reconstructed track and the radar measurement, which is then a point in the three–dimensional space.

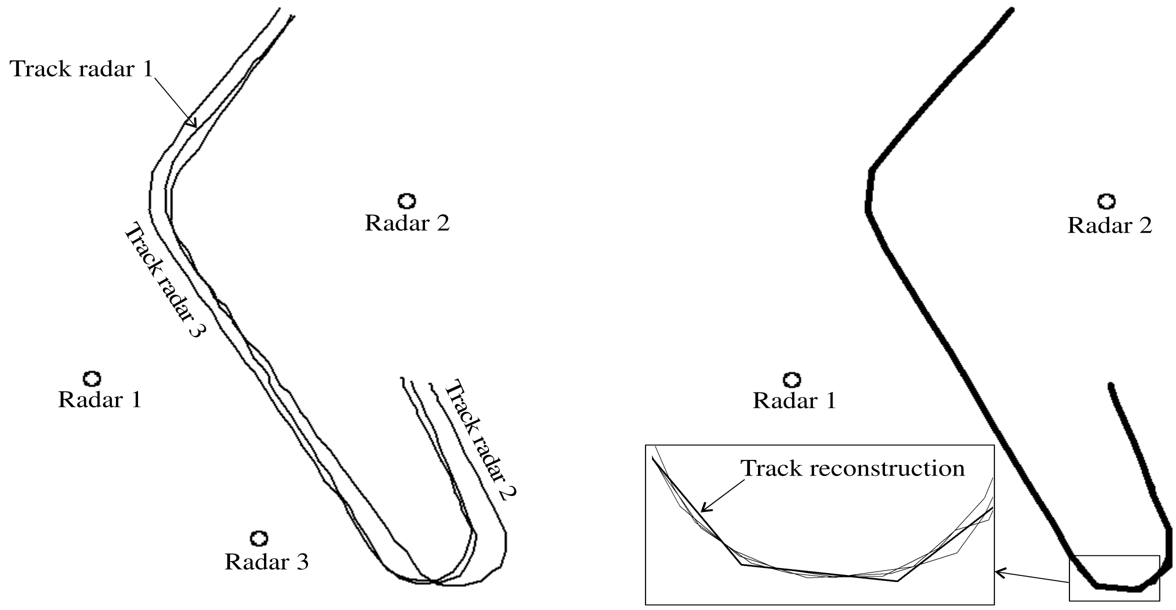
The algorithm was debugged on both model and real data.

For testing the algorithm of finding the radar systematic errors in azimuth, a collection of typical trajectory motions of the aircraft was taken (namely, direct motion, turn, maneuver on take–off and landing in the airport zone, etc.), the coordinates of several radars were given (see the figure), and the ideal measurements of the azimuth, slant range, and the altitude were simulated with necessary time–step. In simulation of the model trajectory motion of the aircraft under observation, the standard system of ordinary differential equations of the airplane motion was used that is adopted for navigational computations [4, 5].

Further, for simulation of real corrupted measurements of each radar, the systematic errors in azimuth and random errors (distributed by the normal law) in the azimuth, slant rang, and altitude were inserted.

The algorithm determines the systematic errors with good accuracy.

An example of finding the systematic errors on the basis of model data is shown in the figure.



Radar tracks; at the left: the initial tracks without corrections in azimuth;
at the right: the tracks turned by the computed systematic errors

For real radar data, stability of reconstructed values of the systematic errors is not always suitable. This can be explained by some factors, which have not been taken into account in the model. For example, such factors can be the systematic error dependence on the radar direction to the aircraft, the systematic errors in the slant range distance, and so on.

The work was performed under Program "Mathematical Control Theory" of RAS and supported by Project 09–P–1–1015 of Ural Branch of RAS and RFBR Grants 09-01-00436, 10-01-96006.

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